

A Hybrid Parallel Method for 3D Nonlinear Periodic Eddy Current Problems with Motions

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This paper describes a highly robust and efficient hybrid parallel computing method for the periodic eddy current problems considering motions. This hybrid parallel is composed of two levels of parallelization with the distributed memory parallel based on MPI (Message Passing Interface) at the higher level and shared memory parallel based on multi-threading at the lower level. Application examples will be presented to demonstrate the effectiveness of this method.

Index Terms—Nonlinear eddy current, Parallel computing, Distributed memory parallel, Shared memory parallel.

I. INTRODUCTION

THE numerical simulations of nonlinear periodic eddy currents have a wide range of low-frequency electromagnetic applications which include actuators, electrical machines, transformers, and so on. There are two typical methods to numerically analyze nonlinear periodic eddy current problems: harmonic balance method and time domain method [1][2]. The harmonic balance method [3] is a very efficient method to deal with the nonlinearity, but it is very difficult to handle motions [2]. In order to handle motions, the physical equations have to be solved in the time domain. In literatures, several methods have been proposed to solve nonlinear periodic eddy currents problems [1][2][4-9] in time domain. In this paper, we propose a hybrid parallel method, which leverages high performance computing for solving nonlinear periodic eddy currents problems in the time domain. Furthermore, we demonstrate its parallel efficiency by examining an application example.

II. PRECONDITIONED ITERATIVE SOLVER

The finite element method discretization of nonlinear periodic eddy current problems produces a semi-discrete form as

$$S(x, t)x(t) + \frac{d}{dt} [T(x, t)x(t)] = f(x, t) + \frac{d}{dt} [w(x, t)] \quad (1)$$

The solution and excitation vectors satisfy the periodic condition

$$x(t) = x(t + \tau), f(t) = f(t + \tau), w(t) = w(t + \tau) \quad (2)$$

with τ the period of the system. Note that $S(x, t)$ and $T(x, t)$ are dependent on the solution vector $x(t)$ to reflect the non-linearity of the eddy current problems. Applying the backward Euler method and the Newton-Raphson method, we have the following linearized matrix equations.

$$\begin{bmatrix} K_1 & 0 & \dots & 0 & M_n \\ M_1 & K_2 & \dots & 0 & 0 \\ 0 & M_2 & \dots & \vdots & \vdots \\ \vdots & \vdots & \dots & K_{n-1} & 0 \\ 0 & 0 & \dots & M_{n-1} & K_n \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_{n-1} \\ \Delta x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix} \quad (3)$$

In the above, $K_i = \Delta t S'_i + T'_i$ and $M_i = -T'_i$ are the Jacobian matrices, Δx_i is the increment of solution during nonlinear iterations, and b_i is the residual during the nonlinear iterations. The submatrix on the right upper corner is due to the property of the periodic condition.

An iterative solver with a preconditioner can be used to solve the block matrix (3). A preconditioner is an approximation to the block matrix (3) such that it can accelerate the convergence of iterations of the iterative solver. There are many ways to construct a preconditioner. For example, reference [8] proposed a preconditioner based on incomplete LU factorizations of a group of submatrices, but the convergence is very slow and it may fail to converge for real engineering problems. In order to accelerate the convergence, we have investigated several preconditioners and found the Gauss-Seidel preconditioner based on the lower triangular part of the block matrix, i.e.,

$$\begin{bmatrix} K_1 & 0 & \dots & 0 & 0 \\ M_1 & K_2 & \dots & 0 & 0 \\ 0 & M_2 & \dots & \vdots & \vdots \\ \vdots & \vdots & \dots & K_{n-1} & 0 \\ 0 & 0 & \dots & M_{n-1} & K_n \end{bmatrix} \quad (4)$$

is overall highly robust and efficient. Also, it naturally facilitates a hybrid parallel scheme, which will be detailed in next section.

III. HYBRID PARALLEL

There are two fundamental methods of parallel computing: distributed memory parallel and shared memory parallel. This proposed method leverages hybrid parallel computing, which uses both distributed memory parallel and shared memory parallel. This hybrid parallel scheme method is composed of two levels of parallel with the distributed memory parallel based on MPI (Message Passing Interface) at the higher level and shared memory parallel based on multi-threading at the lower level, as indicated in Fig.1. The entire nonlinear transient simulation is divided into a number of groups along the time-axis with one MPI process handling one group. For each MPI process, the shared memory

parallelization can be further applied for submatrix assembling, factorizations and postprocessing for each time step within one group.

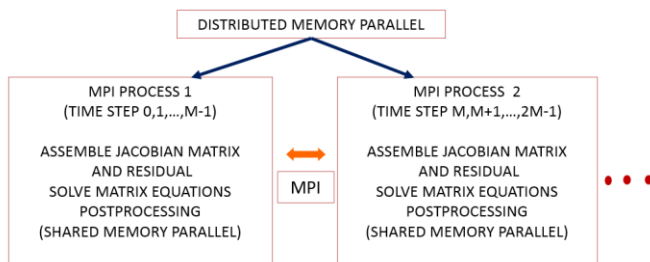


Fig. 1. Hybrid parallel. High level: Distributed memory; Low level: Shared memory.

IV. APPLICATION EXAMPLE

The proposed method has been applied to the simulations of different types of electrical machines, transformers and other magnetic devices. As a typical example, Fig. 2 shows a three phase synchronous machine with nonlinear-magnetic materials for both rotor and stator. The speed is 1000 RPM and driven frequency is 50 Hz. One period is divided into 128 time steps. The number of mesh elements is 388659. The number of non-linear iteration is 18, and the number of iterations of iterative solver is only 2 or 3 for each nonlinear iteration with setting The tolerance of residual of the iterative solver to 10^{-6} . The total number of iterations of iterative solver being 49. Fig.3 shows the calculated torque. It can be seen that the torque profile has entered into steady-state immediately from time 0 due to the constrain of periodic condition. Table I presented the performance of the machine. The speedup is evaluated against two MPI processes. In this case, the speedup using distributed memory with a single thread is 4.7 for 16 MPI process. It can be further increased to 11 using hybrid parallel with 16 MPI processes and 8 threads per MPI process. More numerical results will be discussed in the full paper.

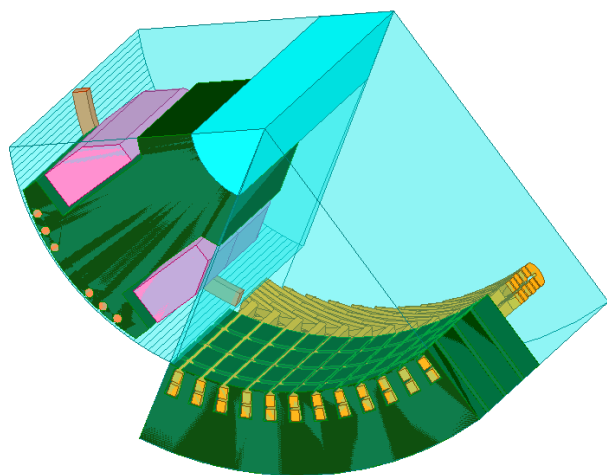


Fig.2. Three phase synchronous machine.

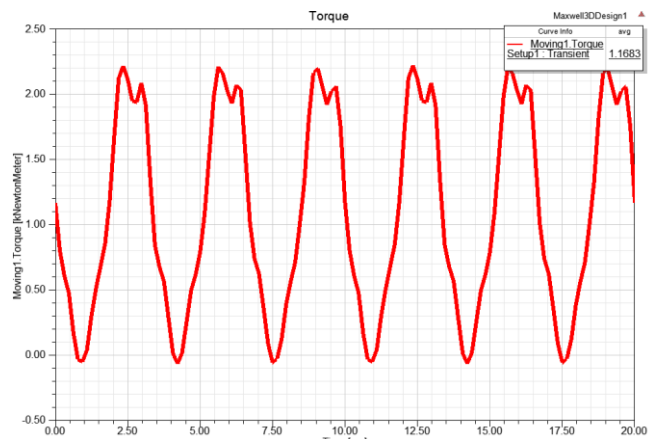


Fig.3. The steady-state torque profile of three phase synchronous machine.

TABLE I
PERFORMANCE OF HYBRID PARALLEL

Number of MPI processes	Number of threads per MPI process	Simulation time (hours:minutes:seconds)	Speedup
2	1	(59:47:01)	1
4	1	(33:09:26)	1.8
8	1	(19:49:10)	3.0
16	1	(12:47:29)	4.7
16	2	(08:47:47)	6.8
16	4	(06:25:58)	9.3
16	8	(05:34:25)	11

REFERENCES

- [1] J. Pries, H. Hofmann, "Steady-state algorithms for nonlinear time-periodic magnetic diffusion problems using diagonally implicit Runge-Kutta methods", *IEEE Trans. Magn.*, vol. 51, no. 4, pp. 1-12, Apr. 2015.
- [2] J Pries, *Computationally Efficient Steady-State Simulation Algorithms for Finite-Element Models of Electric Machines*, Ph.D. dissertation, University of Michigan., 2015.
- [3] S. Yamada, K. Bessho, J. Lu, "Harmonic balance finite element method applied to nonlinear AC magnetic analysis", *IEEE Trans. Magn.*, vol. 25, no. 4, pp. 2971-2973, Jul. 1989.
- [4] O. Biro, K. Preis, "An efficient time domain method for nonlinear periodic eddy current problems", *IEEE Trans. Magn.*, vol. 42, no. 4, pp. 695-698, Apr. 2006.
- [5] T. Hara, T. Naito, J. Umoto, "Time-periodic finite element method for nonlinear diffusion equations", *IEEE Trans. Magn.*, vol. 21, no. 6, pp. 2261-2264, Nov. 1985.
- [6] R. Albanese, E. Coccoresse, R. Martone, G. Miano, G. Rubinacci, "Periodic solutions of nonlinear eddy current problems in three-dimensional geometries", *IEEE Trans. Magn.*, vol. 28, no. 2, pp. 1118-1121, Mar. 1992.
- [7] D. Zhong, H. Hofmann, "Steady-state finite-element solver for rotor eddy currents in permanent-magnet machines using a shooting-Newton/GMRES approach", *IEEE Trans. Magn.*, vol. 40, no. 5, pp. 3249-3253, Sep. 2004.
- [8] Y. Takahashi et al., "Parallel time-periodic finite-element method for steady-state analysis of rotating machines", *IEEE Trans. Magn.*, vol. 48, no. 2, pp. 1019-1022, Feb. 2012.
- [9] M. Clemens, M. Wilke, T. Weiland, "3-D transient eddy-current simulations using FI 2 TD schemes with variable time-step selection", *IEEE Trans. Magn.*, vol. 38, no. 2, pp. 605-608, Mar. 2002.